AE3610 Experiments in Fluid and Solid Mechanics

DISPLACEMENT MEASUREMENTS OF STRUCTURE SURFACES USING DIGITAL IMAGE CORRELATION

Objective

One object of this experiment is to explore the displacement and strain behavior of structures using the digital image correlation technique. In addition, you will explore the interesting response of polypropylene, a material that exhibits different moduli in tension and compression. The experiment consists of two tests: 1) a four point bending test of a polypropylene specimen from which you will estimate the elastic moduli of polypropylene; and 2) a tension test of a second polypropylene specimen with a hole cut in it, from which you can determine the strain and stress concentrations caused by the hole. The loading for both tests will be accomplished using an Instron load frame.

Background

Digital Image Correlation

In the context of structural testing, Digital Image Correlation (DIC) is a method for tracking the point-wise displacements of a structure (typically a surface of the structure) using a series of images of the structure undergoing deformation. DIC is a non-intrusive measurement technique since nothing has to be mounted to the specimen directly. Furthermore, DIC can measure real structural component geometries in real world conditions. The DIC measurements are primarily limited by image resolution, such that higher resolution images produce more accurate results. Alternatively, a higher-resolution displacement field can be captured by zooming the camera’s field of view to a smaller portion of the specimen of interest. To use DIC, it is usually necessary to prepare the specimen by painting a high-contrast speckle pattern on the surface so that subsequent pre- and post-deformation images can be analyzed to accurately determine the displacement field on the structural surface.

There are some requirements and rules-of-thumb for producing good speckle patterns. First, the pattern should be “random”; a highly organized and repeatable pattern would
produce ambiguity in the displacement measurement. Second, the size of the speckles (or high contrast objects) should be roughly the same as (or slightly larger than) a 3×3 pixel region on the digital image for optimal tracking of the displacement. If the speckles are smaller than this, the image magnification can be changed to meet this criterion. Finally, the “density” of speckle features should be sufficient to have an average of 3-4 such features in a 10×10 pixel region. DIC does not rely on correlating a single speckle features, but rather multiple features to get an average displacement in a (multi-pixel) sub-region of the image.

DIC has its origin in speckle imaging approaches used in solid mechanics, and correlation-based analysis methods developed in the 1980’s for object tracking in image processing applications and particle-based velocimetry measurements in fluid mechanics. In fact, DIC is very similar to a common velocity-field measurement approach used in fluid mechanics, Particle Image Velocimetry (PIV). In both DIC and PIV, the individual displacements of many small subregions of an imaged area are obtained by comparing images before and after the displacement has occurred. For each subregion, the “before” (pre) and “after” (post) images are cross-correlated, sometimes using Fast Fourier Transform (FFT) algorithms. The displacement for that subregion is the one that provides the best correlation between the two images. In DIC, the displacement is the desired quantity. In PIV, this displacement is divided by the (short) time between the two images to obtain the local velocity.

This analysis process is typically performed after recording a sequence of images. In DIC, after the displacement field is calculated, the strain field can be determined. The two-dimensional surface displacement field is characterized as $u(x,y)$ and $v(x,y)$, where $u$ and $v$ are the displacements in the $x$ and $y$ directions for a point originally at location $(x,y)$. With $u$ and $v$ determined, we can obtain the surface strain field using the strain-displacement relationships

$$
\varepsilon_x = \frac{\partial u}{\partial x} ; \varepsilon_y = \frac{\partial v}{\partial y} ; \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}.
$$

(1)

Note that differentiating the displacement data amplifies the noise in the data; so advanced analysis software like the package used here\(^1\) employ additional processing approaches such as sophisticated smoothing to find the strain field from the displacement data.\(^2\)

\(^1\)In this lab, we will use the Aramis analysis software package.
\(^2\)You will also export the Aramis DIC data so you can analyze the full-field data.
DIC (and PIV) systems come in different flavors. For example, the measured displacements of a thin region can be two-dimensional (2D) or three-dimensional (3D). In this lab, we will use a 3D DIC system (shown in Figure 1) that enables us to capture displacements in all three coordinate directions, including out-of-plane deformations. A 3D system provides more information than the more common 2D systems by adding an additional measurement. The 2D system requires only one camera (or equivalently, only camera imaging viewpoint). To capture out-of-plane deflections, the 3D DIC system uses two cameras in a stereoscopic configuration. It is important to reiterate that 3D DIC (and PIV) systems provide three components of displacement (or velocity) from a surface (or thin planar region). There are also volumetric DIC and PIV approaches that provide 3D results for each location within a three-dimensional volume.

Our 3D DIC system, which employs two 5-megapixel cameras, will measure the displacements within a test volume that is centered on the test specimen. While the 3D DIC system provides accurate 3D displacement fields, it requires additional calibration effort compared to a simple, single-camera 2D system. The cameras are calibrated by orienting a thermally balanced plate with calibrated markings on it within the test volume.¹

**Four-Point Flexure Testing**

The four-point flexure or bending test is designed to test the flexural response of a slender beam. The goal of the four point bending test is to create a state of pure bending. Pure bending is a stress state where the bending moment is constant and the shear resultant is zero everywhere. The diagram on the left in Figure 2 illustrates a pure bending condition in which only opposing bending moments are applied to either end of the beam. Unfortunately, it is difficult to generate a pure bending moment in a real experiment. Instead, we will use opposing off-set point loads that generate a couple at either end of a beam. This configuration is shown on the right diagram in Figure 2. The advantage of this loading condition is that over the central span there are no shear loads and the beam is subject only to a bending moment.

Figure 3 illustrates the setup of the test apparatus for the four point bending test. The test specimen is placed on rollers which are placed below and above the specimen. The load frame applies a compressive load to the experimental apparatus which transmits point loads to the

¹The Aramis software will guide you through the calibration.
top and bottom of the beam. Rollers are used to ensure that simple support conditions are imposed.

**Under ideal conditions in pure bending**, the strain in the beam should be linear through the thickness:

$$
\varepsilon_x = -\kappa y .
$$

(2)

Furthermore, **if the material is linear elastic and isotropic**, then Hooke’s law applies and the bending moment can be calculated as follows:

$$
M = -\int_A \sigma_x y dA = -\int_A E \varepsilon_x y dA = \int_A E \kappa y^2 dA = EI\kappa .
$$

(3)

where $I$ is the second moment of area of the beam, which is given as

$$
I = \frac{wh^3}{12} .
$$

(4)

In Eq. (4), $w$ is the width of the beam, and $h$ is its depth. **Therefore, if we impose $M$ through a four point bending test, and can estimate $\kappa$ from the digital image correlation results, we can infer the elastic modulus from**

$$
E = \frac{M}{I\kappa} .
$$

(5)

Polypropylene, however, exhibits different elastic moduli under tension and compression. As a result, the neutral surface is not at the geometric centroid of the cross-section and we have to use a composite beam analysis technique to find the elastic modulus under tension and compression. The measured strain will be offset from the geometric centroid, i.e., Eq. (2) has to be modified as follows:

$$
\varepsilon_x = -\kappa y - b ,
$$

(6)

where $b$ is a strain offset.

The through-thickness stress distribution is shown in Figure 5. The $y$-location of the neutral surface can be found as $y_N = -b/\kappa$. The bending moment can be found by a similar integration as that in Eq. (3), except using Eq. (6) for the strain and integrating separately on either side of the neutral surface, i.e.,

$$
M = -\int_{-w/2}^{w/2} \int_{-h/2}^{h/2} -E_T (\kappa y^2 + by) dy dx - \int_{-w/2}^{w/2} \int_{y_N}^{h/2} -E_C (\kappa y^2 + by) dy dx .
$$

(7)
Integrating Eq. (7) yields

\[ M = wE_T \left( \kappa \left( \frac{y_N^2}{3} + \frac{h^3}{24} \right) + \frac{b}{2} \left( \frac{y_N^2}{4} - \frac{h^2}{4} \right) \right) + wE_c \left( \kappa \left( \frac{h^3}{24} - \frac{y_N^3}{3} \right) + \frac{b}{2} \left( \frac{h^2}{4} - \frac{y_N^2}{4} \right) \right). \] (8)

Substituting our expression for \( y_N \) in (8) and simplifying gives

\[ M = wE_T \kappa \left( \frac{h^3}{24} + \frac{b^3}{6\kappa^3} - \frac{bh^2}{8\kappa} \right) + wE_c \kappa \left( \frac{h^3}{24} - \frac{b^3}{6\kappa^3} + \frac{bh^2}{8\kappa} \right) \]

\[ = \frac{w}{24} E_T \kappa \left( h - \frac{2b}{\kappa} \right)^2 \left( h + \frac{b}{\kappa} \right) + \frac{w}{24} E_c \kappa \left( h + \frac{2b}{\kappa} \right)^2 \left( h - \frac{b}{\kappa} \right). \] (9)

Next, we know from equilibrium that there cannot be an internal axial load in the beam. Therefore, the axial resultant \( (N) \) must be zero:

\[ N = 0 = \int_A \sigma_y dA \]

\[ = \int_{w_2}^{w_2} \int_{h_2}^{h_2} -E_T (\kappa y + b) dy dx + \int_{w_2}^{w_2} \int_{y_N}^{y_N} -E_C (\kappa y + b) dy dx . \] (10)

Integrating Eq. (10) and replacing \( y_N \) with \(-b/\kappa\) yields

\[ 0 = E_T \left( \frac{2b}{\kappa} - h \right)^2 - E_c \kappa \left( \frac{2b}{\kappa} + h \right)^2 . \] (11)

Now combining Eq. (11) and Eq. (9) for the bending moment, eliminating \( E_C \) and using our expression for the 2\textsuperscript{nd} moment of area, Eq. (4), results in

\[ M = \frac{wh}{12} E_T \kappa \left( h - \frac{2b}{\kappa} \right)^2 = E_T \kappa \left( 1 - \frac{2b}{h\kappa} \right)^2 . \] (12)

Eq. (12) can be solved for the modulus under tension, giving

\[ E_T = \frac{M}{I\kappa \left( 1 - \frac{2b}{h\kappa} \right)^2} . \] (13)

Inserting this into Eq. (11) provides an expression for the compressive modulus

\[ E_C = \frac{M}{I\kappa \left( 1 + \frac{2b}{h\kappa} \right)^2} . \] (14)

**Stress Concentrations**

Stresses around defects and sudden changes in a structure can be significantly higher than the average stress in the structure. These sharp increases in stress are called stress
concentrations. A good example of a stress concentration is the behavior of the stress near a circular hole in a structure subject to uniform tension or compression. For an infinite plate loaded in-plane, the tangential stress around the edge of the hole has an analytic solution

$$\sigma_{\theta\theta} = \sigma_{av}(1 - 2\cos 2\theta)$$

(15)

where the coordinate $\theta$ runs circumferentially around the circular hole and $\sigma_{av}$ is the average stress in the specimen far away from the hole. The direction $\theta=0$ is aligned with the loading direction, or the $x$-axis in our geometry. The maximum value of the stress (due to the hole) normalized by $\sigma_{av}$ is known as the stress concentration factor.

The stresses in planar cylindrical coordinates (i.e., $\sigma_{rr}$, $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$) can be calculated from Cartesian stresses (i.e., $\sigma_x$, $\sigma_y$ and $\gamma_{xy}$) using standard coordinate transformations, for example

$$\sigma_{rr} = \cos^2 \theta \sigma_x + 2 \sin \theta \cos \theta \gamma_{xy} + \sin^2 \theta \sigma_y$$

$$\sigma_{\theta\theta} = \frac{1}{r^2} \left( \sin^2 \theta \sigma_x - 2 \sin \theta \cos \theta \sigma_{xy} + \cos^2 \theta \sigma_y \right).$$

(16)

In this lab, you will measure displacements and strains in a polypropylene test specimen (Figure 6) with a circular cutout subject to tension. The DIC system will enable us to visualize the distribution of strains around the hole and observe the stress and strain concentration.

**Preliminary**

*The following items must be turned in at the start of your lab session.*

1. Find values for the modulus of elasticity and the Poisson's ratio of polypropylene (try [www.matweb.com](http://www.matweb.com)). Report both upper and lower bounds for the modulus of elasticity.

2. Assuming an Instron loading of 150 lbf, estimate the maximum axial strain, $\varepsilon_x$, and maximum axial stress, $\sigma_x$ (in lbf/in$^2$), expected in the beam under 4-point loading using Euler-Bernoulli beam theory. To simplify this estimation, you may assume that the elastic modulus is the same in tension and compression (big hint: review Eq. (5)). Assume a rectangular beam cross-section of depth $h$ (in inches) and width $w$ (in inches), i.e., the results of your calculation can be on a per $w$ and/or per $h$ basis.

3. Bring a list giving the Instron loading value you would use for each of the tests required in the procedure (but within the described limits) – with a short explanation of why you chose those values.
Procedure

Normally an important step in a DIC experiment is to prepare the surface to be measured. Proper preparation of the surface is crucial to obtaining high quality data. For this lab, the specimens have already been prepared, so the steps below are for your enlightenment.

- First the surface was sanded using sandpaper on a sanding block to remove any loose material particles produced from cutting the specimen. Only the rough edges were sanded and as little material was removed as possible. The bars were then wiped clean.

- Next, a very light coat of white paint was applied to one entire cut side. The smooth sides should not be painted. The paint was allowed to dry for 2 minutes, then another light coat of paint was applied. This procedure was repeated a few times until the white paint coats the specimen evenly and completely.

- Finally, a toothbrush is used to apply black paint. The toothbrush is dipped into the black paint and excess paint is tapped off. The toothbrush is used to “spray” black paint onto the white painted surface in a “uniform” distribution of small speckles. The “spraying” is repeated until the specimen is covered with ~50% black speckles.

1. Calibrate the 3D DIC system using the Aramis software, which will lead you through the calibration procedure. During and after the DIC system calibration, it is very important that the cameras are not moved - or even touched, otherwise the entire calibration procedure will have to be repeated.

2. Measure the thickness and width of the beam for the four-point bending test.

3. Measure the thickness and width of the open hole test specimen, and the hole diameter.

Four-Point Bending Test

4. Position the beam in the test fixture using a 12-inch lower support length and a 4-inch upper support length. Place the roller locations to produce the four-point bending parameters listed in Table 1.

5. Take an un-loaded reference image, and then a second unloaded image.

6. Use the Aramis software to select your target area and perform an analysis. Extract the full field information. If you are satisfied with your results, move on – if not, you may need to recalibrate.

7. Take an un-loaded reference image.

8. Apply the load to the Instron test fixture. Choose a load not exceeding a total of 166 lbf.

9. Take the deformed image and unload the specimen.
10. Use the Aramis software to select your target area and perform the analysis. Extract the full field information.

11. If you are satisfied with your results, remove the specimen from the Instron.

Open-Hole Tension Test

12. Position the specimen with the hole in the test fixture.

13. Take an un-loaded reference image, and then a second unloaded image.

14. Use the Aramis software to select your target area and perform an analysis. Extract the full field information. If you are satisfied with your results, move on.

15. Take an un-loaded reference image.

16. Load the test specimen with a load not exceeding 2.5 kN.

17. Take the deformed image and unload the specimen.

18. Use the Aramis software to select a target area and perform the analysis. Extract the full field information for the area selected.

19. If you are satisfied with your results, repeat steps 15-18 for a second loading value that is lower than your previous measurement.

20. If you are satisfied with your results, remove the specimen from the Instron.

**Data to be Taken**

1. Thickness and width of the beam, and load value used in the four-point bending test.

2. Displacement and strain data from the Aramis software for the four-point bending test.

3. Thickness and width of the specimen, hole diameter, and load value used in the open-hole tension test.

4. Two sets of displacement and strain data from the Aramis software for the open-hole tension test.

**Data Reduction**

1. Using a best linear fit for the appropriate region of your $\varepsilon_i(y)$ plots from the four-point bending test, compute the slope ($\kappa$) and intercept ($b$) relative to a coordinate axis centered on your specimen.

2. From your experimental data, determine the tensile and compressive moduli ($E_T$ and $E_C$) for polypropylene.
3. From your plots of transverse normal strain $\varepsilon_y$ divided by the axial strain $\varepsilon_x$, determine a value for Poisson’s ratio ($\nu$) for polypropylene.

4. For the open-hole tension test, compute the stresses in your specimen based on the measured strains and the appropriate measured modulus of elasticity for polypropylene.

Results Needed for Report

1. Table of beam dimensions and load value used in the four-point bending test.

2. Table of specimen dimensions, hole size and load value used in the open-hole tension test.

3. “Image graphs” or “full-field color plots”\(^1\) of the axial (horizontal) and transverse (vertical) displacement field for the field of view analyzed by the Aramis software in the bending test. Choose your color scaling wisely to accentuate any important gradients. Be sure to also include color bars showing how your colors map to the values of displacement.

4. Images of $\varepsilon_x$, $\varepsilon_y$, and $\gamma_{xy}$ for the field of view imaged and analyzed by the Aramis software in the bending test, with color bars.

5. For the four-point bending test, plots of the axial and transverse normal strain fields as a function of $y$ (with the y-axis as defined in Figure 5), at three axial locations between the two inner load points (i.e., within the constant moment region). One of these should be the center axial location.

6. Plot of the transverse normal strain $\varepsilon_y$ divided by the axial strain $\varepsilon_x$ along the center line used for Result 5.

7. Table of measured values of $\kappa$, $b$ and the polypropylene properties $E_T$, $E_C$ and $\nu$. Include in your table the published values for the polypropylene properties (include your source for those values).

8. Images of the axial strain ($\varepsilon_x$) and transverse strain ($\varepsilon_y$) fields for the region analyzed by the Aramis software in the open-hole tension test, with color bars, for both loadings.

9. Images of the axial ($\sigma_x$) and transverse stress ($\sigma_y$) fields based on your measured strains and measured modulus of polypropylene, with color bars, for both loadings.

10. Images of the normal radial stress ($\sigma_{rr}$) and normal tangential stress ($\sigma_{\theta\theta}$) fields for the open-hole tension tests, with color bars, for both loadings.

\(^1\)Here an “image” means a false-color image, where each displacement (or strain) measurement location is like a pixel in the false-color image, and the color of the pixel corresponds to the value being shown (axial displacement in this case). You can do this, for example, with the `image` function in Matlab.
11. A single graph containing plots of the tangential stress along a line perpendicular to the length dimension of your specimen that passes through the center of the hole, for each loading.

12. A single graph containing plots for each of the loadings of the tangential stress (normalized by its value far from the hole) as a function of angle around the hole, close to the edge of hole.

**Table 1.** Four-point loading parameters to be used in experiment (defined in Figure 2).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$ (in.)</td>
<td>3</td>
</tr>
<tr>
<td>$s$ (in.)</td>
<td>4</td>
</tr>
<tr>
<td>$L$ (in.)</td>
<td>10</td>
</tr>
</tbody>
</table>

**Figure 1.** The 3D DIC imaging systems, with two cameras mounted in a stereoscopic configurations and two light sources.
Figure 2. Idealized pure bending load (left); four point bending test load (right).

Figure 3. Apparatus schematic for a four point bending test.
Figure 4. Photograph of experimental setup for four point bending test.

\[ \varepsilon_x = -y \kappa - b \]

\[ \sigma_x = \begin{cases} 
  \frac{E_T}{E_c} \varepsilon_x & \varepsilon_x \geq 0 \\
  \frac{E_c}{E_c} \varepsilon_c & \varepsilon_x < 0 
\end{cases} \]

Figure 5. Through-thickness stress-strain distribution for material with different moduli in compression and tension.

Figure 6. Photograph of test specimen with hole.