AE2610 Introduction to Experimental Methods in Aerospace

DYNAMIC RESPONSE OF A 3-DOF HELICOPTER MODEL

Objectives

The primary objective of this experiment is to introduce the student to the measurement and response of a dynamical system. The experiments will occur on a tandem-rotor helicopter model connected to a pinned, swivel arm with a counter weight. Optical shaft encoders sense the motions of the helicopter model. The students will calibrate the thrust control (voltage) input to the DC motors that control the rotors. Then the students will measure the response of the system, specifically elevation, to various thrust inputs. Finally, the system response will be compared to a dynamical model of the helicopter.

Background

a) Dynamical Systems

A dynamical system is one in which the effects of an action do not achieve their impact immediately; the dynamical system evolves with time. The time evolution of the system depends both on the time-history of the external input(s) to the system and the properties of the system itself. Dynamical systems exist in many fields and applications:

- when a pilot changes the throttle (thrust) setting, the vehicle’s velocity changes only gradually – but the lighter the vehicle, the faster it can reach the final velocity;
- when a cook puts something in the oven, its temperature does not rise to the oven temperature instantly – and larger items heat more slowly than smaller dishes;
- when a company lowers the price of a product, the impact on the product’s sales (and the company’s stock) can take months to evolve – and that one input is not the only thing that effects the revenues (or stock price).

This way of looking at a system is in contrast to static approaches. For example in the previous labs, you looked at the response of a system to an external input: strain response of a system to a given stress, or lift on a wing as a function of angle-of-attack and wind speed. In those instances, however, we did not consider any rate issues; we assumed there was a unique (unvarying) relationship between the inputs and outputs of the systems. This was because we ensured that the rate at which we applied the changes to the inputs (or how long we waited to
acquire the data) was much slower (or longer) than how fast the system took to reach a steady-state, i.e., how long it took to stop changing.

A powerful way to understand or predict the behavior of a dynamical system is through mathematical models. The models include variables that describe the current state of the system, the state variables, and the things driving the system, i.e., the external inputs. For example, we can look at a point mass that is free to move, the state variables would be: position ($x$), velocity ($v$) and acceleration ($a$), with three components of each if the mass can move in three dimensions. As you learned in physics for simple 1-d motion, these variables are not independent, i.e., $a = dv/dt$ and $v = dx/dt$.

As an example of how one develops a model, we begin with a simple mass $m$ pushed by an external force and constrained to move in one dimension. In this case, the motion of the mass is governed by Newton’s Law, i.e.,

$$F_{external} = ma = m\frac{d^2 x}{dt^2} \Rightarrow m\ddot{x}(t) = F_{external}(t) \quad (1)$$

where the notation $\dddot{\cdot}$ represents the second derivative with respect to time. Now consider what happens if we add a spring that anchors the mass to a wall. Some of the force applied to the mass may now have to compress or expand the spring. You probably also learned in physics that the spring force can be modeled by Hooke’s Law (Eq. 1),

$$F_{spring} = -kx$$

where $k$ is the spring constant, and $x=0$ is taken as the point where the spring is neither compressed or expanded. Then the acceleration of the mass attached to the spring would be modeled according to Eq. 2.

$$m\dddot{x} = F_{external} + F_{spring} \Rightarrow m\dddot{x}(t) + kx(t) = F_{external}(t) \quad (2)$$

We can also add another device, called a damper, that produces a force that always opposes the motion of the mass – and the opposing force is proportional to the velocity, i.e.,

$$F_{damper} = -bv = -b\dot{x}$$

where $b$ is the damping coefficient and $'$ means a first derivative with respect to time. Now including all the forces acting on our mass, we have

$$m\dddot{x}(t) + b\dot{x}(t) + kx(t) = F_{external}(t)$$

or equivalently

$$\dddot{x}(t) + \frac{b}{m}\dot{x}(t) + \frac{k}{m}x(t) = \frac{F_{external}(t)}{m} \quad (3)$$

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This **differential equation** models the dynamics of what is known as a **spring-mass-damper** system, which is illustrated in Figure 1. This simple system represents a number of real systems; for example, the suspension systems used on cars combines springs and shock absorbers (dampers). Similarly, an electrical circuit composed of a capacitor, inductor and resistor in series has the same dynamics. Moreover, this simple system displays behaviors that are also seen in more complex systems, like our helicopter model. For example, if we remove the damper from the system, i.e., set the damping constant to essentially zero, the mass would continuously move back and forth as the spring is repeatedly compressed and expanded. On the other hand if the damping was very high, it would be very hard to move the mass quickly. Also note that the coefficient of the last term on the left side ($k/m$) has units of $1/s^2$ or a frequency squared.

b) **3-DOF Helicopter Model**

The 3 degree-of-freedom (DOF) helicopter mechanism used in this experiment is shown in Figure 2. The 3-DOF helicopter consists of a base upon which an arm is mounted (see Figure 3). The arm carries the helicopter body at one end and a counterweight at the other. Two DC motors with rotors mounted on the helicopter body generate a force (which we will call **thrust**) based on the voltages applied to each of the motors. The portion of the thrust generated by the rotors that is in the vertical direction can cause the helicopter body to lift off the ground, because the arm is held by a pin-type connector. The purpose of the counterweight is to reduce the power requirements on the motors. The standard counterweight location is normally adjusted such that applying about 7-8 V to the motors results in hover with the arm in a horizontal position.

The whole arm can **pitch**, which allows the elevation of the model to change, and it can also travel (yaw) in an azimuthal direction around the central swivel shaft. Optical encoders mounted on these axes allow for measuring the pitch and travel of the model. The helicopter model is mounted at the end of the arm as shown in Figure 4, and the helicopter body (or essentially the arm between the rotors) is free to roll about the arm. A third encoder measures this roll angle. All electrical signals to and from the main swivel arm are transmitted via a slip-ring with eight contacts, thus eliminating the possibility of tangled wires and reducing the amount of friction and loading about the moving axes.

In this experiment, the two rotors will be driven by a **control system** with a Matlab/Simulink™ interface that will use feedback control to keep the model from traveling/yawing or from rolling. Thus, you will consider only the **pitch** (or elevation) and total **lift** (produced by both rotors) as a function of time.
To determine the steady-state condition of our helicopter model, we can examine a static moment balance on the arm. From Figure 5, we can see that the model will not change its pitch angle $\theta$ (or equivalently the elevation of our model helicopter) when the moments about the pinned arm are balanced, i.e.,

$$F_{cw}l_h - (F_h - F_L)v_x = 0$$

where the forces are the components in the vertical direction and the distances are measured in the horizontal direction. This expression incorporates the assumption that the mass of the lever arms are negligible. We can then solve for the lift force produced by the rotors/fans,

$$F_L = \frac{F_h l_a - F_{cw} l_h}{l_a}.$$  \hspace{1cm} (4)

Note, the lift force is only a component of the total thrust, since the fans only point directly downward at zero pitch (i.e., when the model is in a horizontal position).

When the lift produced by the rotors satisfies Eq. 4, the helicopter will have the correct thrust to hover at the given pitch angle. If we change the thrust, the helicopter will have a new steady-state hover condition; after changing the thrust, however, it will take some time for the helicopter to get to the new steady-state pitch/elevation.

You will be running three different experiments on the helicopter model.

1. Calibrating the motor control voltage, i.e., determining the thrust produced by a given motor control voltage.
2. Measuring how the system responds when its controller is told to move the helicopter to a given pitch angle ($\approx 20^\circ$), but with different levels of system damping.
3. Measuring how the system responds when you provide a thrust control input that is a sudden increase in thrust that stays constant (also known as a step input).

In addition, you will be using Matlab to simulate the dynamics of the helicopter model for comparison to your experimental data.

c) Optical Shaft Encoders

The motions to be measured in this experiment are all related to the rotational motion of a shaft. Thus the Quanser system employs shaft (or rotary) encoders to measure the pitch, travel and roll of the helicopter model. Generally, a shaft encoder converts an angular position to an analog or digital output. While there are a number of electro-mechanical approaches employed
in encoders, the most common employ either magnetic or optical sensors to read the shaft motion, with the latter more common, especially in high resolution encoders. These types of systems have replaced older electronic potentiometer-based systems.

Optical shaft encoders typically rely on the rotation of an internal code disc that has a pattern of opaque lines or shapes imprinted on it. The disc is rotated in a beam of light, e.g., from a small LED, and the pattern of dark and light regions on the disc act as shutters, blocking and unblocking the light in systems based on transmission, or strongly and weakly reflecting the light (see Figure 6). For high resolution, the disc is divided into many segments, typically in concentric rings. Internal photodetectors sense the alternating intensity of the light and the encoder's electronics convert the pattern into an electrical output signal.

There are two basic types of encoders: absolute and relative (also called incremental). An absolute encoder returns an exact position, or an angle in a rotary encoder, relative to a fixed zero position. An incremental encoder provides information on the instantaneous movement of the device, which can be converted to speed, distance traveled or position relative to an arbitrary initial state. Absolute encoders are necessary if a particular angular position must be known or retained, independent of when the encoder was powered on. Most other applications can employ an incremental encoder. Absolute encoders provide a output with a unique code pattern that is derived from independent tracks on the encoder disc which correspond to individual photodetectors and represents each position. Incremental encoders are simpler and provide information about the instantaneous motion of a rotating shaft by determining how many and/or how fast the alternating disc patterns go by.

**PRELIMINARY**

*The following must be turned in at the start of your lab session.*

1. Develop an expression, by starting with Eq. 4, that gives the thrust force required to produce steady hover as a function of a given pitch angle ($\theta$); there should be no variables in your expression except for $\theta$; the other system parameters you need can be found in Table 1. When you turn this expression in, include a short development of your expression starting with Eq. 4.
**EXPERIMENTAL PROCEDURE**

*Please note the helicopter model can be damaged if you apply too much thrust – for this reason, please carefully check all inputs to the helicopter control system before operating it.*

*Always stay clear of the helicopter while running and catch it while stopping*

Calibration of rotor motor control voltage

1. Make sure the counterweight is connected to the endmost attachment hole (the hole farthest from the helicopter); we will call this hole 1.

2. Measure the moment arm ("horizontal" distance, e.g., \(l_{cw}\) in Figure 5) between the arm’s pivot point and the location of holes 1, 4 and 7, where you are going to place the counterweight (hole 4 would be the 4th hole from the end).

3. Turn on both PA-0103 Power Modules for the helicopter.

4. On the control computer, begin the Simulink software by double-clicking on the C:\AE 2610\Helicopter 2610\Helicopter_Thrust_Identification.slx file. This opens a Simulink window showing the controller block diagram. Note, it may take a few seconds after Matlab starts for the window to pop up. Click the **Build Model** button to generate the code required to run the controller. Look in the Matlab command window and wait for the status to indicate the model is *downloaded to target*; the controller is now ready to run. Navigate back to the block diagram window and set the **run time** to 45 seconds (in the editable text box).

5. Have someone support the arm of the system (on the helicopter side of the center support) to keep the helicopter level (at the horizontal position); THEN click the **Connect to Target** button in the menu bar, followed by clicking the **Run** button. Once the rotors have starting turning, you can let go of the arm. The person running the computer should watch the timer and warn the person catching the helicopter to be ready. **NOTES:** 1) if the helicopter does not hover at the horizontal position, you may have to do this step again, but this time hold the helicopter a little higher than the horizontal position; 2) if you get an error message when you hit run, you will need to go back and click **Build Model** again and do this step over.

6. Double click the voltage output module (green) to show a plot displaying the rotor motor input voltage as a function of time. Click the **Autoscale** button to resize the plot. Pink values show the filtered and averaged motor voltage.
7. In the command window enter the following commands (note Filename1 and Filename2 are names you pick):

```
csvwrite('Filename1.csv',motor_voltage_f)
csvwrite('Filename2.csv',time).
```
This saves the filtered motor voltage and time values to a your chosen filename. Make sure that the two filenames are different. Check that the data has been saved, and the file can be opened, and then move the file to your group’s folder.

8. Move the counterweight to the second location and repeat steps 5-7. When you remove and reinsert the bolt back in the counterweight, make sure to do this without stripping the threads. It will help if you hold the counterweight firmly against the arm while you are unscrewing and screwing the bolt.

9. Move the counterweight to the third location and repeat steps 5-7.

10. Close the block diagram window - but **DO NOT save the model when prompted.**

11. Make sure the counterweight is replaced in hole 1.

**Response of helicopter/controller for different damping levels**

12. Using the current folder window in Matlab, open the *Helicopter_Damping_Ratio.slx* file. In the command window enter: \( K_d = # \) where # is a value that determines the damping of the system (a higher value corresponds to more damping). The first time you do this, pick a value in the range 30–50.

13. Click the **Build Model** button to generate the code required to run the controller. Look in the Matlab command window and wait for the status to indicate the model is *downloaded to target*; the controller is now ready to run. Navigate back to the block diagram window and set the **run time** to 45 seconds.

14. As before, keep the helicopter horizontal/level THEN click the **Connect to Target** button, and then the **Run** button. Let go of the arm once the rotors begin turning. The controller is going to move the helicopter to a positive pitch angle (~20°), hold it there for a short time, and then return it to zero pitch for a short time until the run is over. **REMEMBER** to have someone ready to catch it when the time is up.

15. Double click the green **voltage display** and **pitch display** modules to see how they each vary with time.

16. Save the following variables/data: **motor_voltage_f**, **time**, and **theta**.

17. Enter a new value of Kd in the range 70-100, and repeat steps 14-16.
18. Close the block diagram window - but **DO NOT save the model when prompted.**

**Response of helicopter to step input to motor voltage**

19. Open the *Helicopter_thrust_input.slx* file. In the command window, enter \( T = \# \) where \( \# \) is a value between 0.2 and 0.4. Then enter \( K_v = 19.23 \).

20. Click the **Build Model** button to generate the code required to run the controller. Look in the Matlab command window and wait for the status to indicate the model is downloaded to target; the controller is now ready to run. Navigate back to the block diagram window and set the **run time** to 60 seconds.

21. As before, support the arm to keep the helicopter horizontal/level, click the **Connect to Target** button and then click the the **Run** button. Let go of the arm once the rotors begin turning. **REMEMBER** to have someone ready to catch it when the time is up.

22. Save the following variables/data: *motor_voltage_f*, *time*, and *theta*.

**DATA TO BE TAKEN**

1. Lever arm distances for the counterweight positions.
2. Motor control voltages required to produce horizontal hover for three locations of the counterweight.
3. Measurement of the helicopter’s pitch and thrust control voltage versus time for two damping values.
4. Measurement of the helicopter pitch and thrust control voltage versus time for the step rise in the thrust control voltages.

**DATA REDUCTION**

1. Using a modified moment balance like Eq. 4, find the **thrust force** required to produce horizontal hover for each of your counterweight locations.
2. Use a regression analysis (e.g., a least-squares fit) to find a relationship between the lift force and the motor control voltages.
3. Convert your measurements of pitch angle and control voltage versus time to helicopter elevation and thrust versus time.
SIMULATION PROCEDURE (to be performed sometime after the lab period)

1. Double click the *pitch_thrust_input_model.slx* file (available on the class web site) to open the Simulink model. This opens a Simulink window showing the controller block diagram. Note, it may take a few seconds after Matlab starts for the window to pop up.

2. Ignore the warning text that may be displayed in the command window.

3. In the command window, enter $T = \text{val}$ where $\text{val}$ is the thrust value in Newtons you determined from the the experiment corresponding to your experimental procedure steps 19-22 based on the thrust calibration from steps 1-9.

4. Click the Run button in the block diagram window.

5. Save the following variables/data: $\text{time}$, and $\theta$.

RESULTS NEEDED FOR REPORT

1. A plot of thrust as a function of voltage, and the linear relationship between the thrust and the motor control voltages.

2. A plot of the helicopter model’s applied thrust and its elevation as a function of time for the two damping cases.

3. A plot of the helicopter model’s thrust and its elevation as a function of time for the step input; the plot should include both the experimental and Matlab model results.

Table 1. Helicopter parameters (see Figure 5 for parameter definitions).

<table>
<thead>
<tr>
<th></th>
<th>0.63 m</th>
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</thead>
<tbody>
<tr>
<td>$l_h$</td>
<td></td>
</tr>
<tr>
<td>$m_{cw}$</td>
<td>1.87 kg</td>
</tr>
<tr>
<td>$\frac{F_{h}l_{h} - F_{cw}l_{cw}}{l_{h}}$</td>
<td>0.445 N*</td>
</tr>
</tbody>
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*Based on “flight” configuration for lab (not calibration configuration).
Figure 1. Schematic of one-dimensional spring-mass-damper system.

Figure 2. Photograph of Quanser 3 degree-of-freedom helicopter mechanism in GT lab.
Figure 3. Schematic of 3-DOF helicopter mechanism.

Figure 4. Main parts of the 3-DOF helicopter mechanism (looking from the back and top).

- Counterweight
- Elevation encoder
- Terminal block
- Shaft
- Slipring
- Bearing block
- Motor connectors
- Encoder connectors
- Travel encoder
- Front
- N/A
- Back
- Travel
- Pitch
- Elevation
- Roll encoder
- Roll positive
- Screw base down to taste if necessary
Figure 5. Free body diagram for helicopter model, with $m_{cw}$=counter weight mass, $m_h$=helicopter model mass, $l_b$=arm length from arm’s pin location to the center-of-mass of helicopter “body”, $F_T$=thrust force, $F_L$=lift force.

Figure 6. Schematic of basic transmissive (top) and reflective (bottom) approaches used in standard optical encoders.